First observational constraints on tensor non-Gaussianity sourced by primordial magnetic fields from cosmic microwave background

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Primordial magnetic fields (PMFs) create a large squeezed-type non-Gaussianity in tensor perturbation, which generates non-Gaussian temperature fluctuations in the cosmic microwave background (CMB). We for the first time derive an observational constraint on such tensor non-Gaussianity from observed CMB maps. Analyzing temperature maps of the WMAP 7-year data, we find such tensor non-Gaussianity is consistent with zero. This gives an upper bound on PMF strength smoothed on 1 Mpc as $B_{1\ \rm Mpc} < 3.2$nG at 95% CL. We discuss some difficulties in constraining tensor non-Gaussianity due to spin and angle dependence of resultant CMB bispectrum.

I. INTRODUCTION

Primordial non-Gaussianity is a powerful probe of inflationary models and various aspects of its property, e.g., amplitude and scale dependence, have been investigated from a diversity of cosmological and astrophysical observables. To date, methods to estimate parameters characterizing non-Gaussianity in primordial perturbations from the cosmic microwave background (CMB) have been extensively investigated by many authors [1–8]. Some specific types of non-Gaussianity have already been constrained by observed data, e.g., $f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8$, $f_{\rm NL}^{\rm eq} = -42 \pm 75$ and $f_{\rm NL}^{\rm orth} = -25 \pm 39$ (68% CL) [9] (For scale-dependent non-Gaussianities, see Ref. [10]).

These bounds have been estimated under an assumption that the primordial non-Gaussianity arises from the scalar perturbations. On the other hand, there exist various models for the early Universe which predict non-Gaussianities associated with not only scalar mode but also vector and tensor modes [11–16]. Despite that many attempts have been made so far to constrain primordial non-Gaussianities, those in vector and tensor perturbations predicted by these models have yet to be constrained so far. Provided predictions from theoretical models and precise data from current CMB observations, we believe it is timely to investigate constraints on non-Gaussianities in perturbations other than scalar ones. Among various theoretical models, we in this paper focus on the electromagnetic field in the early Universe as a mechanism to generate vector and tensor non-Gaussianities [17, 18].

By cosmological observations of galaxies, cluster of galaxies and cosmic rays, the existence of large-scale magnetic field at the present Universe is supported (See e.g. Refs. [19, 20]). There have been a number of studies in which vector fields which exist during inflation are examined as sources for the observed magnetic field [21–24]. However, so far there exist no viable models for the magnetogenesis via primordial vector fields [25–29]. While these theoretical considerations strongly restrict model-building, phenomenological approaches to constrain primordial magnetogenesis are also important. Specifically, through impacts on the CMB anisotropy, properties of primordial magnetic fields (PMFs) which are assumed to be generated from primordial vector fields can be constrained. For example, observational constraints on the amplitude of PMFs as well as its scale-dependence can be obtained by the CMB power spectra alone (For current bounds, see e.g. Refs. [30–33]).

Assuming that field strength of PMFs has a Gaussian distribution, its anisotropic stress fluctuation creates both the scalar and tensor squeezed-type non-Gaussianities due to the quadratic dependence on the field strength. This leads to non-Gaussian CMB anisotropy and suggests that higher order correlation functions or polyspectra of CMB anisotropy beyond the power spectrum should also be informative in probing PMFs. On the basis of this concept, this paper newly explore an observational constraint on the PMF strength by evaluating the magnitude of non-Gaussianity in the CMB temperature anisotropy.

In the case of PMF, the tensor non-Gaussianity dominates over the scalar one [34] and hence the non-Gaussian temperature fluctuation mainly holds information of the tensor mode. Since CMB fluctuations generated from PMFs have unique features and are distinct from those from ordinary scalar perturbations in inflationary Universe, non-trivial constraint can be expected to be obtained. In this sense, this work corresponds to a first attempt to constrain tensor non-Gaussianities from CMB data. This is also another motivation of this paper.

This paper is organized as follows. In the next section, we discuss how to simulate non-Gaussian temperature maps generated from PMFs. In Sec. III, we estimate limits on tensor non-Gaussianities from PMFs using the observed temperature maps of the WMAP 7-year result and translate them into observational bounds on the

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PMF model. The final section is devoted to summary and discussion.

II. NON-GAUSSIANITY IN THE CMB MAP GENERATED FROM PRIMORDIAL MAGNETIC FIELDS

First, we shall briefly summarize the mechanism of PMFs to generate CMB temperature fluctuations and its signatures in the observed CMB maps. After PMFs are produced and stretch beyond horizon during inflation, the anisotropic stress of PMFs acts as a source term in the Einstein equation and supports the growth of curvature and tensor perturbations even on superhorizon scales until neutrino decoupling. However, subsequent to neutrino decoupling, finite anisotropic stress fluctuation in neutrino cancels out the magnetic anisotropic stress fluctuation and therefore the enhancement of the metric perturbations ceases. The resultant curvature and tensor perturbations produce the CMB scalar and tensor anisotropies, which are called passive mode fluctuations [35]. Supposing that the PMFs are quantum-mechanically created and the probability distribution of their field strength obeys pure Gaussian statistics, the PMF anisotropic stress fluctuation becomes a highly non-Gaussian quantity due to the quadratic dependence on the Gaussian PMF strength. This anisotropic stress fluctuation has the local-type shape and therefore the bispectrum of resultant metric perturbations is amplified in the squeezed limit. Consequently, CMB anisotropies generated from PMFs should be non-Gaussian and their bispectrum can be nonzero [17, 18, 34]. The CMB temperature anisotropy for given direction $\hat{\mathbf{n}}$ is quantified via the spherical harmonics expansion as $\frac{\Delta T(\hat{\mathbf{n}})}{T} = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$. If the rotational invariance is preserved, the CMB bispectrum is expressed as

$$\left\langle \prod_{n=1}^{3} a_{\ell_n m_n} \right\rangle = \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{\ell_1 \ell_2 \ell_3} \ . \quad (1)$$

Note that $B_{\ell_1\ell_2\ell_3}$ is proportional to the magnetic field strength to the sixth power. According to Ref. [34], in the case of the PMF, the tensor perturbation dominates over total signal of the CMB bispectrum for $\ell < 500$, and the contributions of the scalar and vector perturbations are small. Therefore, in this paper, we consider that the CMB bispectrum in Eq. (1) is generated only from tensor non-Gaussianity.

To extract the information of the PMFs from the observed CMB maps, we need to simulate $a_{\ell m}$ from the theoretical CMB bispectrum. According to Refs. [36–38], given a power spectrum C_ℓ and bispectrum $B_{\ell_1\ell_2\ell_3}$, a random realization of CMB temperature anisotropy $a_{\ell m}$ should be given as

$$a_{\ell m} \equiv a_{\ell m}^{\rm G} + a_{\ell m}^{\rm NG} \ , \quad (2)$$

$$a_{\ell_1 m_1}^{\rm NG} = \frac{1}{6}\left[\prod_{n=2}^{3} \sum_{\ell_n m_n} \frac{a_{\ell_n m_n}^{\rm G *}}{C_{\ell_n}}\right] \times \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{\ell_1\ell_2\ell_3} \ . \quad (3)$$

Here, $a_{\ell m}^{\rm G}$ is the Gaussian part of the realization whose variance is given by $\left\langle \prod_{n=1}^{2} a_{\ell_n m_n}^{\rm G} \right\rangle = C_{\ell_1} (-1)^{m_1} \delta_{\ell_1,\ell_2} \delta_{m_1,-m_2}$, and $a_{\ell m}^{\rm NG}$ denotes the non-Gaussian part of the realizations.

Eq. (3) indicates that to generate a single realization, a direct implementation requires $\mathcal{O}(\ell_{\rm max}^5)$ arithmetics, where $\ell_{\rm max}$ is the maximum multipole. For $\ell_{\rm max} \sim 1000$, required computational time is enormous. In the literature [36–38], the reduced form of $a_{\ell m}^{\rm NG}$ has been found for the case of the standard scalar non-Gaussianity where the angle-dependence is removed. In the CMB bispectrum from such non-Gaussianity, the dependence on (ℓ_1, m_1), (ℓ_2, m_2) and (ℓ_3, m_3) can be separated from one another. Therefore, in this form, the number of summations can be reduced. Unlike the scalar perturbation, the tensor perturbation is a spin-2 angle-dependent quantity. In the tensor case, this property induces extra correlations between ℓ_1, ℓ_2 and ℓ_3 in the CMB bispectrum, which is not seen in the case of scalar bispectrum. In other words, we do not have a reduced from of Eq. (3) and hence need to straightforwardly perform $\mathcal{O}(\ell_{\rm max}^5)$ arithmetics in computation of $a_{\ell m}^{\rm NG}$. Fortunately, in the case of the PMF model, the signal-to-noise ratio of the magnetic tensor mode is saturated at $\ell \simeq 100$ and the summations up to $\ell = 100$ are enough [34], which prevents us from taking too much computational time.

In what follows, we obey the conventional parametrization for the power spectrum of PMFs as

$$\left\langle B^i(\mathbf{k}) B_j(\mathbf{k}') \right\rangle = (2\pi)^3 \frac{P_B(k)}{2} P^i{}_j(\hat{\mathbf{k}}) \delta(\mathbf{k} + \mathbf{k}') \ , \quad (4)$$

$$P_B(k) = \frac{(2\pi)^{n_B+5} B_{1\ \rm Mpc}^2}{\Gamma(\frac{n_B+3}{2})(\frac{2\pi}{1{\rm Mpc}})^{n_B+3}} k^{n_B}, \quad (5)$$

where $P^i{}_j(\hat{\mathbf{k}}) \equiv \delta^i{}_j - \hat{k}^i \hat{k}_j$, n_B and $B_{1\ \rm Mpc}$ are the divergence-free projection tensor, the PMF spectral index and the PMF strength smoothed on 1Mpc scale, respectively. Assuming the generation of the PMFs at the GUT scale and nearly scale-invariant shape of the PMF power spectrum as $n_B = -2.9$, the 1σ error for a cosmic variance limited survey with $\ell_{\rm max} = 100$ is evaluated as $\delta B_{1\ \rm Mpc} = 3.1$ nG from the Fisher matrix analysis [34].

In the next section, we shall constrain the amplitude of bispectrum given by $A \equiv (B_{1\ \rm Mpc}/3.1\ {\rm nG})^6 \propto B_{\ell_1\ell_2\ell_3}$. Note that theoretically, A should take positive values. Then, the expected 1σ error above corresponds to $\delta A = 1$. Later we will confirm that the error estimated from actual observational data is slightly relaxed compared with this value.

III. OBSERVATIONAL LIMITS

Given a theoretical template of $B_{\ell_1\ell_2\ell_3}$ for a specific theoretical model, we can in general construct an optimal estimator of the amplitude of the bispectrum of primordial perturbations [4, 39]. An optimal cubic estimator for the amplitude of bispectrum should be given as

$$\hat{A} = \frac{1}{\mathcal{N}}\frac{1}{6}\sum_{\ell_n m_n}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix}\hat{B}_{\ell_1\ell_2\ell_3} \times\left[\tilde{a}_{\ell_1 m_1}\tilde{a}_{\ell_2 m_2}\tilde{a}_{\ell_3 m_3} - 3\mathcal{C}^{-1}_{\ell_1 m_1,\ell_2 m_2}\tilde{a}_{\ell_3 m_3}\right], \quad (6)$$

where $\hat{B}_{\ell_1\ell_2\ell_3}$ is the template bispectrum for the PMF model normalized with $A = 1$, $\mathcal{C}_{\ell_1 m_1,\ell_2 m_2}$ is the total (signal+noise) covariance of observed anisotropy $a_{\ell m}$ and $\tilde{a}_{\ell m} \equiv \sum_{\ell' m'}\mathcal{C}^{-1}_{\ell m,\ell' m'}a_{\ell m}$ is a filtered map. $\mathcal{N}$ is the normalization which guarantees the unit response of the estimator to true A. In addition, $1/\mathcal{N}$ corresponds to the variance of the estimator in the limit of Gaussian perturbations, which therefore gives an error of A.

Summary of our analysis is as follows. We use the foreground-cleaned temperature maps from the WMAP 7-year observation at V and W bands [40, 41][1] with a resolution $N_{\rm side} = 512$ in the HEALPix pixelization scheme [42].[2] To reduce effects of residual foregrounds, we apply the KQ75y7 mask [41], which cuts 28.4 % of the sky. Filtered map $\tilde{a}_{\ell m}$ is computed based on the method of Ref. [43]. This method also allows to marginalize over contributions of Galactic foregrounds adopting spatial templates of synchrotron, free-free and dust emissions in Ref. [41] as well as the monopole and dipole components. The CMB signal power spectrum C_ℓ is computed using the `CAMB` code [44], assuming a concordance flat power-law ΛCDM model with the mean cosmological parameters from the WMAP 7-year data alone [45]. In order to determine the normalization $\mathcal{N}$, we first generate random realizations of non-Gaussian CMB maps with $A = 1$ following the method we described in Sec. II. After adding the simulated noise, we then conduct this mock data into the same pipeline as the observed data. The number of MC samples we adopt in our analysis is 100, which is sufficient for converging results.

Here we present our constraints. First we include multipoles up to $\ell_{\rm max} = 100$. Without template marginalization of Galactic foregrounds, we obtain $A = -5.89 \pm 1.06$ at 1σ, and with template marginalization, $A = -2.11 \pm 1.06$. This shows that effects of residual Galactic foregrounds are significant at these angular scales. On the other hand, if we included multipoles up to $\ell_{\rm max} = 400$, the constraints change to $A = -0.91 \pm 1.06$ regardless for template marginalization of Galactic foregrounds.[3] Accordingly, we can conclude that the WMAP data is consistent with null tensor non-Gaussianity induced by PMFs at bispectrum level. Taking into account the above result and a theoretical prior, $A \geq 0$, we find new upper limit on the PMF strength, namely, $B_{1\ {\rm Mpc}} < 3.2$nG at 95% CL. This is consistent with latest bounds from CMB power spectrum data in the SPT or PLANCK experiment [32, 33].

IV. SUMMARY AND DISCUSSION

The origin of the observed large-scale magnetic fields is one of the most important and interesting issues in probe of the early Universe and some researchers seek answers in the inflationary paradigm. In this paper, we have discussed an observational constraint on the seed magnetic field stretched by the inflationary expansion from the analysis of non-Gaussianities in CMB anisotropy. Signal of the Gaussian field strength of the PMF becomes largest on large scales via the enhancement of non-Gaussian tensor perturbation.

We have analyzed the data of WMAP 7-year temperature maps and confirmed no evidence of squeezed-type tensor non-Gaussianity due to PMFs. Constraining the amplitude of tensor non-Gaussianity leads to an upper bound on the PMF strength as $B_{1\ {\rm Mpc}} < 3.2$nG (95% CL). This result is not sensitive to the residual Galactic foregrounds. This value may be improved by considering impacts of polarizations or adopting more accurate data as the PLANCK data [9].

Aside from the issue on the primordial magnetic field, this is a first challenge to constrain the primordial tensor non-Gaussianity from the CMB bispectrum. The tensor CMB bispectrum has a spectral shape quite distinct from the scalar one and leads to non-trivial constraints which have never seen in the scalar case. Unlike the scalar case, the CMB bispectrum composed of the tensor mode has the correlation between ℓ_1, ℓ_2 and ℓ_3 due to the spin and angle dependence associated with the tensor mode. This correlation seems to prevent us from reducing the formula for the simulated $a_{\ell m}$ (3) to the form with lower computational cost, which is derived in the scalar case. This may be disadvantage for the data analysis with higher resolution and some improvement will remain as a future issue.

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[1] http://lambda.gsfc.nasa.gov

[2] http://healpix.jpl.nasa.gov

[3] Note that we here adopt the normalization $\mathcal{N}$ from multipoles up to $\ell_{\rm max} = 100$. This is because according to Ref. [34], we expect $\mathcal{N}$ should be saturated at $\ell \gtrsim 100$. While this expectation may be affected by the inhomogeneity in noise levels and sky cuts, the effects would not be significant since $\mathcal{N}$ at $\ell_{\rm max} = 100$ shows good agreement with the Fisher matrix analysis of Ref. [34], which assumes homogeneity in noise levels.

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